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The non-perturbative $\mathcal{N} = 2$ SUSY Yang-Mills theory from semiclassical absorption of supergravity by wrapped D branes

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Abstract: The imaginary part of the two point functions of the super conformal anomalous currents are extracted from the cross-sections of semiclassical absorption of dilaton, RR-2 form and gravitino by the wrapped D5 branes. From the central terms of the two point functions anomalous Ward identity is established which relates the exact pre-potential of the $\mathcal{N} = 2$ SUSY Yang-Mills theory with the vacuum expectation value of the anomaly multiplet. From the Ward identity, WDVV (Witten-Dijkgraaf-Verlinde-Verlinde) equation can be derived which is solved for the exact pre-potential.

Keywords: String Duality, Brane Dynamics in Gauge Theories, AdS-CFT and dS-CFT Correspondence, Anomalies in Field and String Theories.

Contents

1. Introduction

Recently the gauge theory/gravity duality, which is commonly known as AdS/CFT duality is extended to non-conformal pure $\mathcal{N} = 1$ or $\mathcal{N} = 2$ supersymmetric Yang -Mills (SYM) theories [1]. So far the perturbative behavior of the $\mathcal{N} = 2$ SYM is produced by this duality. The conventional folklore is that the instantons which are responsible for the non-perturbative part of the pre-potential are suppressed in the large N limit (since the gauge gravity duality is valid only in the large N limit). Since the wrapped D5 branes are far from being conformal, the usual holographic description of bulk-boundary connection is quite subtle to extract all the properties of SUSY Yang-Mills theories. This fact led us to wonder about other ways of getting complete picture of the non-perturbative solution of the SUSY Yang-Mills from gravity. It is a fact in quantum field theory that the absorption/emission cross section of particles is always given by the discontinuities of the two point functions of the currents to which the field couples. To be precise, if the interaction lagrangian is $S_{\text{int}} = \int d^4x \varphi(x)\mathcal{J}(x)$ then the cross section is

$$\sigma = \frac{1}{2i\omega}\mathrm{Disc}\,\Pi(p)\Big|_{\substack{p^0=\omega \\ \vec{p}=0}}\,, \tag{1.1}$$

where

$$\Pi(p) = \int d^4x\, e^{ip\cdot x}\langle\mathcal{J}(x)\mathcal{J}(0)\rangle\,. \tag{1.2}$$

The connection between the central charge of four dimensional energy momentum tensor and the absorption of gravitons/dilatons by the world volume theory of D branes is first explored by Gubser and Klebanov [2]. The trace of the energy-momentum tensor, the γ

trace of the super current and the divergence of U(1) axial current due to the R-symmetry form different components of the superconformal anomalous current $J_{\alpha\dot{\alpha}}$ and its super divergence $\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}}$ is classically zero however possesses anomaly. More explicitly

$$\begin{aligned}
\langle\sqrt{g}\,\theta^{\mu}_{\mu}\rangle &= \frac{1}{2}\frac{\beta(g)}{g^3}\left(F^a_{\mu\nu}\right)^2 + \frac{c(g^2)}{16\pi^2}\left(W_{\mu\nu\rho\sigma}\right)^2 - \frac{a(g^2)}{16\pi^2}\left(\tilde{R}_{\mu\nu\rho\sigma}\right)^2 \\
\langle\partial^{\mu}\sqrt{g}R_{\mu}\rangle &= -\frac{\beta(g)}{3g^3}\left(F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu}\right) + \frac{c(g^2)-a(g^2)}{24\pi^2}\left(\tilde{R}R\right) \\
\langle\sqrt{g}\gamma^{\mu}\mathcal{S}_{\mu}\rangle &= \frac{1}{2}\frac{\beta(g)}{g^3}\left[\left(\sigma_{\mu\nu}\lambda^a F^{a\mu\nu}\right) + \frac{1}{2}R^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}\left(D_{\mu}\psi_{\nu} - D_{\nu}\psi_{\mu}\right) - \left(\epsilon^{\mu\nu\rho\sigma}\gamma_5\psi_{\mu}A^a_{\nu}F^a_{\rho\sigma}\right)\right],
\end{aligned} \tag{1.3}$$

where $\beta(g)$ is the beta function of SYM, $a(g)$ and $c(g)$ are central functions near the criticality, $W_{\mu\nu\rho\sigma}$ is the Weyl tensor and $\tilde{R}_{\mu\nu\rho\sigma}$ is the dual of the curvature tensor and ψ_{μ} is the gravitino field. We first derive the world volume action of the wrapped D5 brane where these currents are minimally coupled to the ten dimensional supergravity fields. Then we calculate the absorption cross-section of these fields by the brane which give us the central functions of these correlators. The organization of this paper is as follows. First we describe briefly the preliminaries of the wrapped D5 brane which gives the $\mathcal{N}=2$ SUSY Yang-Mills theory [9, 10]. Then we show the absorption of dilaton, fluctuating part of RR-2 forms and the gravitino by the world volume of wrapped D5 branes and evaluate the central functions. Consequently we derive the equation relating the pre-potential with the vacuum expectation of the anomaly multiplet and then conclude.

2. The preliminaries

We start with type IIB little string theory which is obtained as a collection of a large number of $NS5$ brane in the vanishing string coupling limit. This gives rise to $D=6$ SYM [8]. Then we dimensionally reduce two of its spatial world volume in such a way that we retain $\mathcal{N}=2$ SYM in the low energy limit [11]. The $NS5$ brane has SO(4) R-symmetry as the normal bundle. When one identifies the U(1) subgroup of the SO(4) R-symmetry with the U(1) spin connection of the two cycle which is compactified, one gets a covariant constant spinor and SUSY is retained, which is commonly known as twisting [1]. This is called wrapping of $NS5$ brane on a supersymmetric two cycles. If the compact space is a two-sphere, then there will be no extra hyper- multiplet and in the low energy limit i.e. in the scale much lower than the radius of the sphere we will get pure $\mathcal{N}=2$ SYM. Thus it amounts to consider a gauged $D=7$ supergravity solution and then lift it to get the solutions in ten dimensions. In recent past many relevant classical solutions have been obtained by authors of ref. [9, 10] which we don't want to repeat instead use their results. The classical solutions of $D=7$ gauged supergravity is properly lifted to ten dimensional string theory. Using S-duality we obtain the metric of the $D5$ brane as

$$\begin{aligned}
ds^2_{10} = \mathrm{e}^{\Phi}\Bigg[&dx^2_{1,3} + \frac{z}{\lambda^2}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) + \frac{1}{\lambda^2}\mathrm{e}^{2x}\,dz^2 + \\
&+ \frac{1}{\lambda^2}\left(d\theta_1^2 + \frac{\mathrm{e}^{-x}}{f(x)}\cos^2\theta_1\left(d\theta_2 + \cos\theta\, d\varphi\right)^2 + \frac{\mathrm{e}^{x}}{f(x)}\sin^2\theta_1\, d\theta_3^2\right)\Bigg],
\end{aligned} \tag{2.1}$$

where the dilaton is

$$e^{2\Phi} = e^{2z}\left[1 - \sin^2\theta_1 \frac{1 + c\,e^{-2z}}{2z}\right] \tag{2.2}$$

and

$$f(x) = e^{x}\cos^2\theta_1 + e^{-x}\sin^2\theta_1\,, \tag{2.3}$$

also

$$e^{-2x} = 1 - \frac{1 + c\,e^{-2z}}{2z}\,, \tag{2.4}$$

where λ is the gauge coupling constant of seven dimensional gauged supergravity and c is a parameter as the integration constant of the classical solution. For $c \geq -1$ the range of the radial variable is $z_0 \leq z \leq \infty$ where z_0 is the solution for $e^{-2x(z_0)} = 0$. Here θ and φ are the angles of compact two-sphere with radius of compactification as $\frac{z}{\lambda^2}$ and θ_1, θ_2 and θ_3 are angles of transverse three-sphere. The conservation of the RR-charge on the transverse sphere S_3 fixes $\frac{1}{\lambda^2} = N\,g_s\,\alpha'$ for large N number of $D5$ branes with string coupling g_s. The R-R- 2 Form is given by

$$C^{(2)} = \frac{1}{\lambda^2}\theta_3\, d\left[\frac{\sin^2\theta_1}{f(x)\,e^{x}}\,(d\theta_2 + \cos\theta\, d\varphi)\right]. \tag{2.5}$$

The $D5$ brane action is given by

$$S = -\tau_5 \int d^6\xi\; e^{-\Phi}\sqrt{-\det\left(G + 2\pi\alpha' F\right)} + \tau_5 \int \left(\sum_n C^{(n)} \wedge e^{2\pi\alpha' F}\right)_{6-\mathrm{form}}\,, \tag{2.6}$$

where F is the world volume gauge field and τ_5 is the brane tension. The BPS condition is fixed from the condition of the vanishing of the potential between two branes which gives θ_1 to be $\pi/2$. This condition makes the transverse boundary of the D brane to be a two dimensional space consisting z and θ_3 which is eventually the moduli space of $\mathcal{N} = 2$ SYM.

3. The absorption of dilaton

To evaluate the central terms for the correlation function of stress tensor we need to calculate the absorption of the fluctuating dilaton coupled to the world volume of the wrapped D5 brane. If we denote the fluctuation of dilaton by η then the world volume action is given by

$$S_{\mathrm{int}} = \tau_5 \int d\Omega_2 \int d^4x e^{-\Phi}\sqrt{-\det G}\;\eta(x,\xi)\left\{\frac{1}{4}F^a_{\alpha\beta}F^{a\alpha\beta} + \frac{1}{2}\overline{\lambda}^a\gamma^\alpha D_\alpha\lambda^a + \frac{1}{2}D_\alpha\overline{\Psi}^a D^\alpha\Psi^a\right\}. \tag{3.1}$$

Here $F^{a\alpha\beta}$ is the field strength of the SUSY Yang-Mills, λ^a are the fermions and Ψ^a are the scalars of $\mathcal{N} = 2$ multiplet in the adjoint representation of the U(N) gauge group. Here $\Psi^a = T^a\, e^{(z+i\theta_3)}$ is the complex scalar which belongs to the transverse space of the $D5$ brane which eventually forms the moduli space of the $\mathcal{N} = 2$ SUSY Yang-Mills vacua and T^a's are the generators of the color U(N) group . It has been shown by Klebanov [4] and Gubser et al. [3] that the string theoretic absorption cross-section and the classical

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absorption cross-section given by the classical equation of motion of dilaton by the $D3$ brane are in exact agreement. The classical absorption probability is given as the ratio of the flux at infinity to the flux where the brane is sitting [5]. The classical equation of motion by the dilaton is

$$\partial_a \sqrt{G} G^{ab} \partial_b \varphi = 0 \,. \tag{3.2}$$

where G^{ab} is the metric given in eq. (2.1). If we define the transverse coordinate $e^z = \rho$ then for large ρ, $G^{\rho\rho} = \rho$ and $\sqrt{G} = \rho^2 R^4$ where R^2 is the square of the compactified volume which is identified with $\frac{1}{\lambda^2}$. The equation of motion for the dilaton is

$$\partial_\rho^2 \varphi + \frac{3}{\rho} \partial_\rho \varphi + \frac{R^2 \omega^2}{\rho^2} \varphi = 0 \,. \tag{3.3}$$

We take the ratio of the flux $\varphi^* G^{\rho\rho} \partial_\rho \varphi$ for $\rho \gg \omega R$ to that for $\rho \leq \omega R$. This is the probability of absorption which multiplied with the proper phase space gives the cross section. This is also same as quantum cross section of the world volume theory by the normalized dilaton fluctuation η c.f. eq. (3.1). The cross -section is

$$d\sigma = \frac{1}{2\omega} \frac{d^5 p_1}{(2\pi)^5 2E_1} \frac{d^5 p_2}{(2\pi)^5 2E_2} (2\pi)^6 \, \delta^6 \left(p_1 + p_2 - q\right) \overline{|\mathcal{M}|}_{\rm fi}^2 \tag{3.4}$$

where $\mathcal{M}_{fi}$ is the matrix element for the dilaton with momentum q going to two gluons, two gluinos and two scalars with momenta p_1 and p_2. We take the vector component of the momenta to be zero and the zeroth component of the dilaton to be ω. The momentum conservation gives $E_1 + E_2 = \omega$. After the integration of the phase space the cross-section is found to be

$$\sigma = \frac{V_2}{16} \frac{\kappa_{10}^2}{32\alpha' \pi} \omega^3 \,, \tag{3.5}$$

where V_2 is the volume of the compact two dimensional space which arises due to the delta function in these directions. Here $V_2 = 4\pi R^2 \ln \rho$. We identify the string coupling $2\pi g_s = g_0^2$, the Yang-Mills coupling. Then

$$\sigma = \frac{\kappa_{10}^2}{32\pi} \omega^3 \left(\frac{g_0^2 N \, \ln \rho}{8\pi^2} \right) . \tag{3.6}$$

This gives the imaginary part of the polarization $\Pi(p)$ (c.f. eq. (1.2)). It has been shown by Anselmi et al.[12] how to get all the central functions of eq. (1.3) from the two point correlation functions. Any two point function of currents

$$\begin{aligned} \mathcal{G}(x) &= \langle \mathcal{J}(x) \mathcal{J}(0) \rangle \\ &= \frac{b(\ln(x\mu))}{x^{2d_0}} \,, \end{aligned} \tag{3.7}$$

where d_0 is the engineering dimension of the current $\mathcal{J}$ and μ is a scale parameter. The two point function of the local operators ought to satisfy the Callan-Symanzik equation,

$$\left(x \frac{\partial}{\partial x} + 2d_0 + 2\gamma_0 + \beta(g) \frac{\partial}{\partial g} \right) \mathcal{G}(x) = 0 \,, \tag{3.8}$$

where γ is the anomalous dimension of the operator and $\beta(g)$ is the beta function of the renormalization group. From eq. (3.6), eq. (3.7) and eq. (3.8) we get

$$\beta(g_{\rm YM}) = -\frac{N}{8\pi^2}\, g_{\rm YM}^3 \,. \tag{3.9}$$

4. The $\mathrm{U}(1)_R$ anomaly

The gaugino and the scalars of $\mathcal{N}=2$ SUSY Yang-Mills theory possess U(1) R-Symmetry. For the wrapped branes it is realized as the rotations in the transverse space mainly by the angle θ_3. However this is periodic and any change of it needs to be discrete like $\frac{\pi}{N}\,k$ since R-R 2-Form flux should be quantized on the vanishing compact two sphere. One realizes the chiral anomaly due to this fact in the gravity dual picture. We want here to calculate explicitly the two point function of the chiral R-current in the world volume theory of wrapped D5 branes. The fermionic partner of the Chern-Simon's term of the second part of eq. (2.6) is

$$S_{\rm int} = \tau_5 \int d^6\xi \;\; \omega_2 \wedge \epsilon^{\mu\nu\lambda\sigma}\, \Pi^a_\mu\, \Pi^b_\nu\, \Pi^c_\lambda\; \overline{\lambda}\; \Gamma_{abc}\;\, \lambda\partial_\sigma c(x)\,, \tag{4.1}$$

where we have used RR-2 form $C^2 = \omega_2\, c(x)$ and $c(x)$ is the fluctuation of C^2. Here $\Pi^a_\mu(x) = \partial_\mu X^a - i\overline{\theta}\Gamma^a\partial_\mu\theta$ is like vielbein in the Green-Schwarz variable and $\{a,b=0,\ldots,10\}$ and $\{\mu,\nu=0,\ldots,3\}$. Integrating over ω_2 and using the properties of the Γ matrices we get

$$S_{\rm int} = N\int d^4x\, \overline{\lambda}i\;\gamma_5\;\gamma_\mu\;\, \lambda\partial^\mu c(x)\,. \tag{4.2}$$

The classical solution of $c(x)$ is θ_3. We want to calculate the semi-classical absorption cross-section of $c(x)$ by the wrapped D5 brane. The classical action for this field is extracted from the low energy limit of the type IIB super string action.

$$\begin{aligned} S = \frac{1}{2k_{10}^2}\int d^{10}x\Big(\sqrt{-g_{10}}\Big[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{12}e^{-\phi}(\partial B_2)^2 - \frac{1}{2}e^{2\phi}(\partial C)^2 - \\ - \frac{1}{12}e^{\phi}(\partial C_2 - C\partial B_2)^2 - \frac{1}{4\cdot 5!}F_5^2\Big]\Big)\,. \end{aligned} \tag{4.3}$$

Here B_n and C_n are n-Form potential due to NS-NS and R-R sectors respectively. We set $C_0 = C$ and F_5 to zero and also $C_2 = -B_2$. Substituting $F^{\theta\,\varphi} = \sin\,\theta$ from eq. (2.5) in eq. (3.3) and integrating over θ and φ coordinates we get the action for $c(x)$ which effectively moves in six dimension as

$$S = \frac{1}{6}\frac{1}{R^2\,\ln\rho_0 k_{10}^2}\int d^6x\left(\sqrt{-g_6}\partial_a c(x)\, g^{ab}\partial_b c(x)\right). \tag{4.4}$$

The classical absorption probability can be calculated as the ratio of the in coming flux for ρ at infinity to the outgoing flux from $\rho < R\omega$. Also from the world volume action the absorption cross-section is

$$\sigma = \int \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\,(2\pi)^4\,\delta^4\,(p_1+p_2-q)\;\overline{|\mathcal{M}|}^2_{\rm fi}\,, \tag{4.5}$$

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where $\mathcal{M}_{\rm fi}$ is given as

$$\mathcal{M}_{\rm fi} = -\frac{1}{2}\sqrt{2\,\tilde{\kappa}_{10}^2} q^\mu \bar{v}(-p_1)(i\gamma_5\gamma_\mu)u(p_2)\,, \qquad (4.6)$$

where $\tilde{\kappa}_{10}$ is the normalization for $c(x)$ given as

$$\tilde{\kappa}_{10}^2 = R^2 \,\ln\rho_0 k_{10}^2\,. \qquad (4.7)$$

This gives the cross-section

$$\sigma = \frac{\kappa_{10}^2}{32\pi} N^2\omega^3 \left(\frac{g_0^2 N\,\ln\,\rho}{24\pi^2}\right). \qquad (4.8)$$

Here again the Callan-Symanzik equation gives the correct β function.

5. The γ trace of the super current and anomaly

The world volume action of the minimally coupled gravitino ψ_μ to the wrapped $D5$ brane is

$$S_{\rm int} = \tau_5 \int d\Omega_2 \int d^4x\, \mathrm{e}^{-\Phi}\sqrt{-\det G}\left\{\frac{1}{4}\psi_\mu\,\gamma^\mu\,\sigma_{\alpha\beta}\lambda^a\;F^{a\alpha\beta}\right\}. \qquad (5.1)$$

The gravitino part of the low energy effective action for the type IIB string theory is

$$S_{\rm gravitino} = \frac{1}{2k_{10}^2}\int d^{10}x\left(\sqrt{-g_{10}}\,\frac{1}{2}\overline{\psi}_\mu\, i\Gamma^{\mu\nu\rho}D_\nu\psi_\rho\right). \qquad (5.2)$$

Due to twisting, the covariant derivative D_ν is like ordinary one here. For the classical equation of motion of the gravitino we choose a gauge $\psi_\mu = \gamma_\mu\chi$ which simplifies the equation of motion for the gravitino as

$$\rho\gamma_5\,\partial_\rho\chi + R\gamma_0\partial_0\chi = 0\,. \qquad (5.3)$$

The probability of absorption is the ratio of the flux $\overline{\chi}\gamma_5\chi$ at $\rho \gg R\omega$ to $\rho < R\omega$. The absorption cross-section from the world volume theory of minimally coupled gravitino gives the same result. The phase space volume which comprise the compact two dimensional sphere gives the $\ln\rho_0$ as the pre-factor which eventually provides the running coupling and the β function. We get the same beta function as the other cases.

6. The exact pre-potential

The low energy effective action for $\mathcal{N} = 2$ SUSY Yang-Mills theory is

$$\Gamma\left(\Phi\right) = \int d^4x d^4\theta \mathcal{F}\left(\Phi\right)\,, \qquad (6.1)$$

where $\mathcal{F}$ is the pre-potential and Φ is the chiral super field of the $\mathcal{N} = 2$ SUSY Yang-Mills theory. To realize the superconformal invariance a supergravity pre-potential $H^{\alpha\dot{\alpha}}$ is coupled to the superconformal current $J_{\alpha\dot{\alpha}}$. The invariance of the effective potential under superconformal transformation up to first order gives the anomalous superconformal Ward

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identity [13]

$$\int d^4x\, d^4\theta\, \frac{\delta\Gamma(\Phi)}{\delta\Phi}\langle\delta\Phi\rangle \;=\; \int d^4x\, d^4\theta\delta H^{\alpha\dot{\alpha}}\langle J_{\alpha\dot{\alpha}}\rangle\,. \tag{6.2}$$

The righthand side of this equation gives $D^{\dot{\alpha}}J_{\alpha\dot{\alpha}}$ which is the divergence of the anomalous superconformal current and its vacuum expectation gives the anomaly,

$$2\mathcal{F} - \mathcal{F}(\mathcal{A})'\mathcal{A} = \frac{N}{8\pi^2}\langle \mathrm{tr}\, v^2\rangle\,, \tag{6.3}$$

where $\mathcal{A}$ is the vacuum expectation value of Φ and trv^2 is the anomaly multiplet for example trF^2 will correspond to θ^{μ}_{μ} etc. Indeed the second order variation of the infinitesimal superconformal parameter ϵ^{α} will relate the two point function of the derivative of the anomalous currents which has been obtained in the last sections to the two point function of the Φ with the derivatives of the pre-potential as the pre factor. This identity is difficult to realize in the coulomb phase when Φ has a vacuum expectation value. So we resort here to the Ward identity for our purpose. We assume here that all the D branes are distributed on a circle with N points in the θ_3 directions. So $\langle\Psi_i\rangle = \rho \mathrm{e}^{i\theta_3^i}$ which is denoted as $\mathcal{A}_i$. In this phase the Ward identity

$$2\mathcal{F} - \sum\nolimits_i \frac{\partial\mathcal{F}}{\partial\mathcal{A}_i}\mathcal{A}_i = \frac{N}{8\pi^2}\langle \mathrm{tr}\, v^2\rangle\,, \tag{6.4}$$

which is same as eq. (1.3) in the flat space. This equation can be rewritten as WDVV equation [7]. This is also same as the renormalization group equation of D' Hoker et al. [14] where the right hand side is given as $\mu\frac{\partial}{\partial\mu}$ of $\mathcal{F}$. To solve for $\mathcal{F}$ the usual ansatz is to assume $\mathcal{F}$ as a genus $N-1$ hyper-elliptic curve,

$$y^2 = P(x)^2 - \Lambda^N \tag{6.5}$$

with canonical homology basis $\{a_i, b_i\}$ and the differential form $\mathcal{S}_{\mathrm{SW}} = \frac{x}{2i}\frac{dP}{\pi\ y}$. The large N solution of this equation was first solved by Douglas and Shenker [15] and recently many subtleties are addressed by Ferrari [16]. The non-perturbative nature of the theory will be studied from the distribution of the roots of eq. (6.5). The singularity occurs when any two of the roots coincide, where a dyon becomes massless. Our moduli space coordinates are $\mathcal{A}_i = \rho\mathrm{e}^{i\theta_3^i}$ and $\theta_3^k = 2\pi\frac{k}{N}$. The difference of $\mathcal{A}_i$ is the W-meson mass and in their vanishing limit mass less particle emerges signaling phase transition. The integration over the homology cycle for example a gives

$$i\pi\,\frac{a}{N} = -\frac{\rho-1}{\rho} + \ln\rho + \frac{1}{N}\left(-\ln\rho + \frac{\rho-1}{\rho}\ln 2 + \frac{(\rho-1)\ln(\rho-1)}{\rho}\right), \tag{6.6}$$

which shows clearly the singularity for $\rho \to 1$. Indeed in our supergravity picture the radius of the wrapped two sphere $z = \ln\rho$ vanishes and signals "enhancons" [6].

7. The conclusion

Here we tried to analyze the non-perturbative effects of $\mathcal{N} = 2$ SUSY Yang-Mills theory starting from the absorption of minimally coupled Supergravity fields by the wrapped $D5$ branes. The absorption cross-section gives the two point correlators of the currents. This is very crucial for our entire analysis which provides the central functions. This encodes the entire evolution of the coupling dictated by the renormalization group. Once we get the beta function correctly we can use it to establish the anomalous Ward identity. It is the consistency of the renormalization group which gives one point function from two point function. This analysis perhaps can be obtained by renormalization group flow of $\mathcal{N} = 4$ to $\mathcal{N} = 2$ [17] which we found to be quite non-trivial.

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